Calculus BC

Section 8.5 - Partial Fractions

(polynomials, factorable denominator)

Recall:

1. $\int \frac{3}{2x+5}\,dx$

2. $\int \frac{3x-15}{x^2-25}\,dx$

Partial Fractions

$f(x)=\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials

Make sure that degree of $P(x)$ < degree of $Q(x)$.

If not, divide first – long division
Factor denominator <u>completely</u>

degree: 0

i) linear factors:

$$\frac{\ldots\ldots}{(x-r_1)(x-r_2)(x-r_3)}=\frac{A}{(x-r_1)}+\frac{B}{(x-r_2)}+\frac{C}{(x-r_3)}$$

degree: 1

ii) quadratic factors:

degree: 1

$$\frac{......}{(x^2-r_1)(x^2-r_2)}=\frac{Ax+B}{(x^2-r_1)}+\frac{Cx+D}{(x^2-r_2)}$$

degree: 2

iii) repeated factors:

keep degree of 0 for repeats

$$\frac{......}{(x-r_1)(x-r_2)^3}=\frac{A}{(x-r_1)}+\frac{B}{(x-r_2)}+\frac{C}{(x-r_2)^2}+\frac{D}{(x-r_2)^3}$$

no repeats repeats

linear factors

3. Change $\frac{5x-3}{x^2-2x-3}$ to partial fractions

-make sure degree of numerator < degree of denominator

-factor denominator completely

$$\frac{5x-3}{(\qquad)(\qquad)}=\frac{A}{(\qquad)}+\frac{B}{(\qquad)}$$

$$=$$

common denominator

let x = ____ to remove A

let x = ____ to remove B

$$\frac{5x-3}{(\qquad)(\qquad)}=\frac{}{(\qquad)}+\frac{}{(\qquad)}$$

optional method: expand to get a system of equation
for A and B

4. Change to partial fractions $\frac{x^2+7x-11}{(x-1)^2(x+2)}$

$$\frac{x^2+7x-11}{(x-1)^2(x+2)}=\frac{}{(\qquad)}+\frac{}{(\qquad)}+\frac{}{(\qquad)}$$

4′. Find $\int \frac{x^2+7x-11}{(x-1)^2(x+2)}dx$